TEMPERATURE DISTRIBUTION IN A GAS CONDUCTOR HEATED BY DIRECT CURRENT

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Approximate solutions are obtained for the energy-balance equation for cylindrical and planar conductors, with an account of the temperature dependence of the electrical and thermal conductivities and the integral radiation. An exact solution is given for a planar conductor.

Cylindrical Conductor (Approximate Solution). A successive-approximation treatment of a cylindrical conductor (an arc column) with an account of radiative energy transfer was reported previously [1-3]. Approximate analytic expressions were obtained for the radiation from an optically thin layer.

The energy-balance equation per unit length of a cylindrical arc column when there is no axial heat flow is written [1]

$$\sigma(T)E^2 - U(T) + \frac{1}{r}\frac{d}{dr}\left[r\lambda(T)\frac{dT}{dr}\right] = 0. \tag{1}$$

Introducing the heat-conduction function $S = \int_0^T \lambda(T)dT$ [4] and the relative radius $\rho = r/R$, we find

$$R^2[\sigma(S)E^2 - U(S)] + \frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dS}{d\rho}\right) = 0. \tag{2}$$

To solve Eq. (2), we divide the cross section of the arc channel into a central conducting region, from $\rho = 0$ to ρ_1 (the boundaries of the conducting region), and a cold region, from ρ_1 to $\rho = 1$, near the wall, in which $\sigma = 0$ and $U = 0$. We assume linear dependences of σ and U on S in the conducting region; then for $\rho_1 \le \rho \le 1$ ($S_W \le S_I \le S_1$), we have

$$\sigma(S) = 0 \text{ and } U(S) = 0, \tag{3}$$

or for $0 \le \rho \le \rho_1$ ($S_1 \le S_{II} \le S_0$), we have

$$\sigma(S) = AS + B \text{ and } U(S) = \alpha S + \beta. \tag{4}$$

Here A, B, α, and β are the linearization constants; and S_W, S_0, and $S_1 = S(\rho_1)$ are the heat-conduction functions at the wall, the axis, and the boundary of the conducting region, respectively.

Using the boundary conditions

$$\rho = 1 \quad S = S_W, \quad \rho = 0 \quad S = S_0,$$
$$\left(\frac{dS}{d\rho}\right)_{\rho=0} = 0 \tag{5}$$

and the joining conditions at the boundary of the linearization regions,

$$\rho = \rho_1 \quad S_I = S_{II}, \quad \left(\frac{dS_I}{d\rho}\right)_{\rho_1} = \left(\frac{dS_{II}}{d\rho}\right)_{\rho_1}, \tag{6}$$

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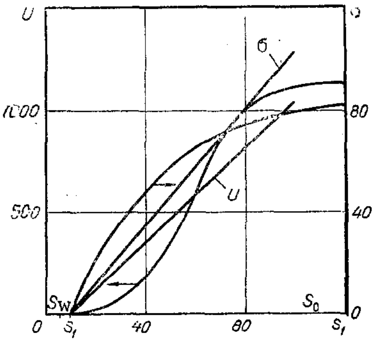

Fig. 1

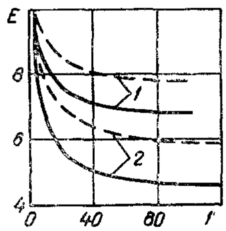

Fig. 2

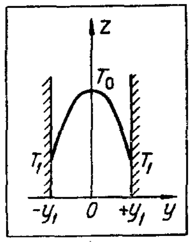

Fig. 3

Fig. 1. Linearization of the properties of an argon plasma with respect to the radius of the arc channel ($p = 9.806 \cdot 10^4\ N/m^2$). Here $U = \alpha S + \beta$ is in W/cm^3, $\sigma = AS + B$, mho/cm, and S is in W/cm.

Fig. 2. Current–voltage characteristics of an arc column in argon ($p = 9.806 \cdot 10^4\ N/m^2$). 1) R = 0.003; 2) 0.006 mm. Dashed curves) With an account of radiation; solid curves) without account of radiation. I is in amperes, and E is in V/cm.

Fig. 3. Temperature distribution in a planar gas conductor.

we find an approximate solution of Eq. (2) in the linearization regions:

$$S_I(\rho) = \frac{S_1 - S_W}{\ln \rho_1} \ln \rho + S_W, \tag{7}$$

$$S_{II}(\rho) = (S_0 - S_1) J_0 \left(\rho R \sqrt{E^2 A - \alpha}\right) + S_1. \tag{8}$$

Here J_0 is the zeroth-order Bessel function of the first kind. The temperature distribution can be found from the S(T) dependence.

The radius of the conducting region is given by

$$\rho_1 = \exp\left\{-\frac{S_1 - S_W}{S_0 - S_1} \frac{1}{\nu_1 J_1(\nu_1)}\right\}, \tag{9}$$

where $\nu_1 = 2.405$ and $J_1(\nu_1)$ is the Bessel function of the first kind. The electric field intensity is

$$E = \sqrt{\frac{\nu_1^2}{\rho_1^2 R^2 A} + \frac{\alpha}{A}}, \tag{10}$$

which converts when $\alpha = 0$ into the familiar Mecker solution of the energy-balance equation without account of radiation: $E = \nu_1/\rho_1 R\sqrt{A}$ [5].

The parameters of the arc column calculated without (subscript 0) and with an account of radiation for identical boundary conditions (S_W, S_0 = const) are related by

$$E = E_0 \sqrt{1 + \frac{\alpha}{E_0^2 A}}. \tag{11}$$

$$I = I_0 \sqrt{1 + \frac{\alpha}{E_0^2 A}}. \tag{12}$$

It follows from Eqs. (7)-(12) that this account of the radiative energy transfer does not affect the radius ρ_1 of the conducting region or the radial temperature distribution, but it does increase the required power by a factor of $(1 + \alpha/E_0^2 A)$.

Figure 1 illustrates the linearization of σ(S) and U(S) for an argon plasma at atmospheric pressure. The σ(S) dependence was taken from [6], and the U(S) dependence was taken from [7]. Figure 2 shows the calculated current–voltage characteristics. The points on these characteristics corresponding to the same column-axis temperature lie on the straight line E/I = const. The calculation was carried out for a maximum temperature of $14 \cdot 10^3$°K at the channel axis. The effect of the radiation increases with increasing channel radius and with increasing current; this leads in turn to an increase in the conduction-region radius. When radiation is taken into account, the similarity condition ER = const at EI = const for arc discharges is disrupted, since the radiation loss is proportional to R^2.

This method of calculating the radial temperature distribution and the current–voltage characteristics, involving the linearization of $\sigma(S)$ and U(S) by a single straight line in the conducting region, gives a satisfactory description of only the descending branch of the current–voltage characteristic; for argon at atmospheric pressure, this corresponds to a temperature T_0 of about $13\text{-}14 \cdot 10^3$°K at the arc-column axis [6]. For hydrogen, the range of applicability of this procedure is much wider, since even at $T_0 = 25 \cdot 10^3$°K the conducting region fills only half the channel [8]. Calculations for an arc column in hydrogen ($p = 9.806 \cdot 10^4$ N/m^2) without an account of radiation, and with a five-region linearization of $\sigma(S)$, show the current–voltage characteristic to be of a descending nature up to an axial temperature of $T_0 = 40 \cdot 10^3$°K. The hydrogen properties were taken from [9].

Planar Conductor (Exact Solution). The energy-balance equation for an optically thin, planar gas conductor (Fig. 3) is written, under the assumptions that there is no energy transfer along the x and z axes and that the current flows along the z axis,

$$\frac{d}{dy}\left[\lambda(T)\frac{dT}{dy}\right] + \sigma(T)E^2 - U(T) = 0. \tag{13}$$

Using the substitution

$$\left(\frac{dT}{dy}\right)^2 = \theta, \tag{14}$$

we convert Eq. (13) to

$$\frac{d\theta}{dT} + \varphi(T)\theta + \psi(T) = 0, \tag{15}$$

where

$$\varphi(T) = \frac{2}{\lambda(T)}\,\frac{\partial\lambda(T)}{\partial T} \text{ and } \psi(T) = \frac{2}{\lambda(T)}[\sigma(T)E^2 - U(T)]. \tag{16}$$

Solving Eq. (15) and using the substitutions (14) and (16), we find the final expression:

$$y = \int\limits_{T}^{T_0} \frac{\lambda(T)\,dT}{\sqrt{C_1 + 2\int\limits_{T}^{T_0}\lambda(T)[\sigma(T)E^2 - U(T)]\,dT}} + C_2. \tag{17}$$

The constants C_1 and C_2 and the electric field intensity E are found from the boundary conditions (Fig. 3). For the symmetric case, we have

$$\begin{aligned} &\text{for } y = \pm y_1 \quad T = T_1, \\ &\text{for } y = 0 \quad T = T_0, \quad \left(\frac{dT}{dy}\right)_0 = 0. \end{aligned} \tag{18}$$

The current flowing through a cross section of the conductor of width $\Delta x = 1$ is

$$I = 2E\int\limits_0^{y_1}\sigma(T)\,dy. \tag{19}$$

Planar Conductor (Approximate Solution). As in the case of a cylindrical conductor, we divide the entire transverse cross section into two regions, and adopt the linear approximation for $\sigma(S)$ and U(S) in Eqs. (3) and (4). Then Eq. (13) becomes

$$\frac{d}{dy}\left(\frac{dS}{dy}\right) + S(AE^2 - \alpha) + (B - \beta) = 0. \tag{20}$$

The solution, for the cold region near the wall, is

$$S_{\mathrm{I}}(y) = C_1 y + C_2 \quad (21)$$

for the conducting region, the solution is

$$S_{\mathrm{II}}(y) = \exp\left[C_4 + \ln(y - C_3) - y\sqrt{AE^2 - \alpha}\right] - \frac{BE^2 - \beta}{AE^2 - \alpha}. \quad (22)$$

Here the constants of integration and E are found from the boundary conditions and from the joining conditions at the boundary of the linearization regions. The σ(S) and U(S) approximation in the conducting region should be replaced by several linear regions to increase the accuracy of the solution.

NOTATION

σ, λ are the electrical and thermal conductivities of the gas current;
U is the integral radiation;
E, I are the longitudinal electric field intensity and current;
T, p are the gas temperature and pressure;
S is the heat-conduction function;
r, R, ρ are the instantaneous radius, channel radius, and dimensionless or relative radius;
y is the transverse coordinate.

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